

Yale University

EliScholar – A Digital Platform for Scholarly Publishing at Yale

Cowles Foundation Discussion Papers

Cowles Foundation

5-1-1966

Estimation of a Fixed Coefficients Vintage Model of Production

Richard Attiyeh

Follow this and additional works at: <https://elischolar.library.yale.edu/cowles-discussion-paper-series>



Part of the [Economics Commons](#)

Recommended Citation

Attiyeh, Richard, "Estimation of a Fixed Coefficients Vintage Model of Production" (1966). *Cowles Foundation Discussion Papers*. 442.

<https://elischolar.library.yale.edu/cowles-discussion-paper-series/442>

This Discussion Paper is brought to you for free and open access by the Cowles Foundation at EliScholar – A Digital Platform for Scholarly Publishing at Yale. It has been accepted for inclusion in Cowles Foundation Discussion Papers by an authorized administrator of EliScholar – A Digital Platform for Scholarly Publishing at Yale. For more information, please contact elischolar@yale.edu.

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

**Box 2125, Yale Station
New Haven, Connecticut**

COWLES FOUNDATION DISCUSSION PAPER NO. 210

Note: Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Requests for single copies of a Paper will be filled by the Cowles Foundation within the limits of the supply. References in publications to Discussion Papers (other than mere acknowledgment by a writer that he has access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

**ESTIMATION OF A FIXED
COEFFICIENTS VINTAGE MODEL OF PRODUCTION**

Richard Attiyeh

May 12, 1966

ESTIMATION OF A FIXED
COEFFICIENTS VINTAGE MODEL OF PRODUCTION*

Richard Attiyeh

I - INTRODUCTION

1. Analysis of the growth and distribution of national income, like most economic problems, requires knowledge of the production possibilities available to society. In order to acquire and apply such technological information, economists have designed analytical constructs that attempt to describe the terms on which aggregate stocks of resources may be converted into aggregate output. These aggregate production functions have been used in the hope that the diverse and complex processes which are observed on the firm level will appear, when viewed in the aggregate, to follow a functional relationship that is both simple and realistic: simple in the sense that it can be manipulated in the context of a complete model and estimated from the available data, and realistic in the sense that it exhibits the characteristics of production essential to the analysis of growth and distribution.

The course of economic growth in recent years has been deceptively favorable to the econometrician who sets himself the task of estimating an aggregate production function. The data that have been generated make it possible to explain the growth of potential output with models that are extremely simple. But since simplicity is obtained in large measure by making crucial untested assumptions about the nature of production it is unclear what interpretation should be given to these results. The apparent statistical success of these models may be due simply to the high collinearity of the data, in which case their structure may be a woefully inaccurate

* This study was supported by the Ford Foundation with a Doctoral Dissertation Fellowship and through the FUSEG project, of which I am a member.

representation of the real world. Alternatively, the untested assumptions may be the appropriate ones to make, and accordingly the models may be highly useful approximations to actual production processes. To determine where, between these two extremes, the truth lies it is essential to construct, test, and estimate more general models.

Two types of simple production functions have been given extensive application. One type of model, which has developed from input-output analysis, assumes fixed input coefficients and has been used primarily in the analysis of growth as a source of instability.¹ The other type of model, which is an outgrowth of the neo-classical theory of production, assumes a positive elasticity of substitution between factors of production and has been used primarily to explain the secular growth of output.²

As indicated above, both of these models, with appropriate allowance for technical change, provide a reasonably good fit to

-
1. See D. Hamberg, "Full Capacity vs. Full Employment Growth" Quarterly Journal of Economics, 1952; R. F. Harrod, "An Essay in Dynamic Economics," Economic Journal, 1953; E. D. Domar, Essays in the Theory of Economic Growth, (New York, Oxford University Press) 1957; and W. Leontief, et al, Studies in the Structure of the American Economy, (New York, Oxford University Press) 1953.
 2. See R. M. Solow, "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, 1956, pp. 65-94, and "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, 1957, pp. 312-320.

aggregate data on potential output, capital and labor.³ Nevertheless, neither model is completely consistent with other available information on the technological facts of life. The observation that in the short-run changes in relative factor prices and cyclical changes in demand induce but limited factor substitution on the micro-economic level suggests that fixed coefficients is the more accurate assumption. Yet, the existence of positive income shares for both capital and labor suggests that a positive elasticity of substitution is the more reasonable assumption. Recognition of this conflicting evidence as to the relative merits of these two simple models has led to several attempts to integrate them in a more general model.

One approach was to formulate a production function with a constant, but unspecified, elasticity of substitution, which would have as special cases the two simple models described above.⁴ For this model the value of elasticity of substitution would not be restricted by assumption, but would be left free to be determined by the data. The preponderance of results from empirical investigation of this model shows industry values of the elasticity of substitution to vary considerably and the overall value to lie between zero and one. Although the

3. For some measures of these series potential output per manhour appears to grow at a constant exponential rate and potential output per unit of capital appears to have no trend. These data are consistent with either a Cobb-Douglas model with disembodied technical change or a fixed coefficients model with Harrod-neutral technical change. (Since deviations from trend for output per manhour are inversely correlated with deviations for output per unit of capital the Cobb-Douglas model performs somewhat better.) The fact that both models seem to explain the data should not be interpreted to mean that they are in any sense equivalent, since they imply widely divergent results from a non-proportional increase in factor inputs.

4. See K. J. Arrow, et al, "Capital-Labor Substitution and Economic Efficiency," Review of Economics and Statistics, 1961, pp. 225-235.

level of statistical significance of these results is quite low,⁵ this model has clearly been useful both by providing useful opportunities for disaggregation by industry and as a means of summarizing the aggregate relationship between potential output and the factors of production. It does not, however, serve to reconcile the divergent views of the production process that obtain when both micro and macroeconomic evidence is considered.

A second approach, which does achieve such a reconciliation, is to construct a model which disaggregates the capital stock into capital types each of which is characterized by different fixed production coefficients. For any unit of new capital no substitution is possible; but, as labor is shifted to new capital from existing capital of another type substitution between aggregate capital and labor does occur.

Although there has been considerable theoretical analysis of this kind of production relation its empirical validity has yet to be tested. The purpose of this study is to provide such a test.

2. Models of production with heterogeneous capital have taken two forms, which differ in the way production coefficients of new capital are determined. One formulation allows for a wide range of possible

5. See M. Nerlove, "Recent Empirical Studies of the CES and Related Production Functions," in forthcoming volume of Studies in Income and Wealth, NBER.

production coefficients for new capital before it is produced.⁶ This menu of new capital types can be summarized by a capacity indifference curve along which the labor requirement of a unit of new capacity varies inversely with the cost of its production. With a given technology, the type of capital that is most profitable to produce will depend on investors' expectations about the course of the real wage rate over the useful life of the capital.

The second formulation of this model, and the one with which this study is concerned, excludes the possibility of ex ante substitution.⁷ In this version the capacity indifference curve is a right angle to the origin and the production coefficients of new capital are independent of present and future wage rates. Given the present state of knowledge, one may only conjecture whether, in fact, opportunities for ex ante substitution exist to any significant degree. It is clear, however, that allowance for ex ante substitution is not essential to obtain all of the results, such as aggregate factor substitution and positive income shares for both factors, that characterize simple neoclassical type production functions. Furthermore, to allow

6. See I. Johansen, "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis," Econometrica, 1959 pp. 157-76; E. S. Phelps, "Substitution, Fixed Proportions, Growth and Distribution," International Economic Review, 1963, pp. 265-288; and R. M. Solow, "Substitution and Fixed Proportions in the Theory of Capital" Review of Economic Studies, 1962, 207-18, and "Heterogeneous Capital and Smooth Production Functions: An Experimental Study," Econometrica, 1963, pp. 623-645.

7. See W. E. G. Salter, Productivity and Technical Change, (Cambridge, 1960) and R. M. Solow, et al, "A Model of Fixed Capital without Substitution," forthcoming Review of Economic Studies.

for ex ante substitution makes both theoretical and empirical analysis of the model much more difficult.

Consider the model in which neither ex ante nor ex post substitution is possible. Assume that technical change takes place, that in part, at least, it must be embodied in new capital goods, and that the embodied portion of technical change is not strictly capital augmenting. This implies that the current vintage of capital can produce no less output per unit but requires less labor per unit than old capital.

The aggregative implications of this model for a competitive economy follow quite simply.⁸ Because of embodied technical progress, newer vintages of capital will be more profitable to operate and the age of the oldest vintage in use will be less than that of any unused vintage. The effect of adding an extra unit of labor to the economy will be to draw into use the newest unused capital, with the resulting increase in output being the marginal product of labor. As additional units are added, capital with higher capital and labor coefficients will be brought into use, and the marginal product of labor will diminish. The effect of adding a unit of new capital will be to draw labor away from the oldest vintage of capital. Since new capital has lower capital and labor coefficients such a reallocation will increase total output, this amount being the marginal product of investment. As more new capital is added labor will be drawn away from increasingly modern vintages of capital and the marginal product of investment will diminish.

8. The following summary follows Solow, et al, op. cit.

The existence of positive but diminishing marginal products of labor and investment suggests two important results. First, the economy can accomodate investment and labor force growth in any proportion.⁹ Second, in the aggregate competition would give both factors of production a relative income share between zero and one. These two results are exactly those which neoclassical production functions were designed to produce, yet they are obtained without assuming either ex ante or ex post substitution.

The advantages derived from using this model go beyond integrating micro and macro production behavior. In particular it provides a more useful device through which production relations can enter into the theory of short-run fluctuations. In this connection existence of different types of capital with fixed coefficients clarifies a number of conceptual difficulties. For example, the meaning of excess capacity becomes simply the capacity of those unused vintages of capital which would be drawn into use if unemployment were eliminated. Also, since new capital is different and better than old capital, it is no longer difficult to explain a positive rate of investment when there is excess capacity or necessary to distinguish between investment for expansion and investment for cost reduction.

3. While the disaggregation of the capital stock into vintages makes possible significant analytical advances, it does create a number of

9. Whatever this proportion may be, the effect of factor growth on the aggregate capital-labor ratio depends on the factor bias of technical change.

econometric problems. The most striking is that, in the absence of data on the economic lives of successive vintages of capital, it is necessary to infer from the date which vintages remained in use at the time of each observation. By excluding the possibility of ex ante substitution, thereby making the production coefficients of each vintage a function of time alone,¹⁰ this problem, though still difficult, becomes manageable.

In fitting the model to data for U. S. manufacturing two kinds of labor (production and nonproduction workers) and two kinds of fixed investment (equipment and plant) are distinguished. This form of disaggregation is suggested by a number of considerations. With respect to labor, the substantial differences in the secular growth and in the skill requirements of these two categories indicate that they are essentially different factors of production. As for investment, differences in time series trends and in observed average lengths of life of plant and equipment also indicate that each serves a different function in the production process.

Of these four factors of production, equipment is assumed to play the key role as the principal vehicle for technical progress. Each vintage of equipment defines a process in which a unit of equipment can produce a fixed rate of output and has fixed requirements for plant and the two types of labor. This scheme is qualified in two ways.

10. To allow for ex ante substitution would make it necessary to infer from the data, not only the rate and bias of technical change, but the elasticity of ex ante substitution and the function of expectations regarding future wage rates, as well.

First, the introduction of disembodied technical change into the model permits these coefficients to change over time, though not at any point in time. And, second, to offset the increased complexity that results from having four factors of production, nonproduction workers and plant are considered to serve as "overhead" labor and capital. That is, it is assumed that the nonproduction worker employment required per unit of output is independent of the vintage of equipment in use and that both the production worker requirement and the output per unit of equipment are independent of the vintage of plant in use. These assumptions make it possible, for estimation purposes, to separate the relationships among output, production workers and equipment from relationships that involve nonproduction workers and plant.

In Chapter II the relationships among output, production workers and equipment are fit to the data. In Chapter III the relationships governing overhead capital and labor are estimated. And, in Chapter IV the factor price implications of the model are considered. Variable definitions and data sources are given in the Appendix.

II - PRODUCTION WORKER AND EQUIPMENT COEFFICIENTS

1. As outlined in Chapter I, the production coefficients of a particular process are determined by the vintage of equipment in use; these coefficients vary between vintages, depending on their relative ages, and over time for each individual vintage; and nonproduction workers and plant enter in such a way that their requirements per unit of output may be considered separately from those for equipment and production workers. In this chapter the equipment and production worker requirements will be considered.

Let $Q(t,v)$ stand for output in period t produced with equipment of vintage v , $N(t,v)$ for production workers in period t using equipment of vintage v , and $E(v)$ for equipment of vintage v . Then

$$(1) \quad Q(t,v) = a(t,v) \cdot E(v)$$

and

$$(2) \quad Q(t,v) = b(t,v) \cdot N(t,v)$$

where a and b represent reciprocals of the equipment and production worker requirements per unit of output for process v in period t . Equation (2), by substitution for $Q(t,v)$ from (1), may be rewritten as

$$(2a) \quad N(t,v) = \frac{a(t,v)}{b(t,v)} \cdot E(v) \quad .$$

By allowing for both a constant rate of improvement in the quality of successive vintages of equipment (embodied technical progress) and a constant rate of increase in the average productivity of production workers and all vintages of equipment over time (disembodied technical progress), equations (1) and (2a) can be replaced with

$$(3) \quad Q(t,v) = a_0(1+g)^v (1+r)^t E(v)$$

and

$$(4) \quad N(t,v) = \frac{a_0(1+g)^v (1+r)^t}{b_0(1+h)^v (1+s)^t} E(v) ,$$

where g and h are the rates of capital and labor augmenting embodied technical progress, r and s are the rates of capital and labor augmenting disembodied technical progress, and a_0 and b_0 are the equipment and production worker coefficients for vintage 0 in period 0 .

Parameters g , h , r and s may be unambiguously interpreted as rates of factor augmentation only when the rate of physical depreciation of equipment is assumed to be zero. If this assumption does not hold, each parameter must be interpreted as a composite of a rate of factor augmentation and a rate of depreciation. The assumption of zero physical depreciation will be made throughout.

If there is efficient production and if g , r , and s are at least zero and h is greater than zero,¹ then aggregation over

1. When embodied technical change is purely capital augmenting efficient production does not require that production be concentrated on the most recent vintages.

vintages of equipment in use results in the following expressions for total output and production workers in period t :

$$(5) \quad Q(t) = a_0(1+r)^t \sum_{t-m(t)}^t (1+g)^v E(v)$$

and

$$(6) \quad N(t) = \frac{a_0(1+r)^t}{b_0(1+s)^t} \sum_{t-m(t)}^t \frac{(1+g)^v}{(1+h)^v} E(v)$$

where $m(t)$ is the age of the oldest vintage of equipment in use.²

2. In order to obtain estimates of equations (5) and (6) it is necessary to have observations on output, production worker employment, and the history of purchases of equipment. For n observations there are $n+6$ parameters to be estimated: six technology parameters (a , b , g , h , r and s) and one m for each observation. Since this leaves $n-6$ degrees of freedom, there must be a minimum of 6 observations for the model to be identified.

Unfortunately, very little data are available for which the assumption of efficient production holds. While there are data for each year from 1947 to 1963, a number of these years have been characterized by large cyclical changes in output. As a number of studies have shown,³ after accounting for secular productivity change, average labor productivity has varied inversely with the unemployment

2. Equations (5) and (6) hold exactly only if all of the oldest vintage equipment is in use. When this condition is not met $E(t-m(t))$ is to be interpreted as the amount of the oldest equipment in use.

3. See, for example, E. Kuh, "Cyclical and Secular Labor Productivity in U. S. Manufacturing," Review of Economics and Statistics, 1965, pp. 1-12.

rate -- a pattern which is inconsistent with the assumption of efficient production for a vintage model with constant returns to scale and labor augmenting technical change.

One explanation for this phenomenon is that hiring and firing costs encourage firms to hoard labor when production is reduced as a result of temporary decline in the demand for output. This hypothesis suggests that during recession periods the quantity of labor employed exceeds the quantity of labor actually engaged in production. A second explanation is that during periods of cyclical change in output the allocation of labor among vintages is less efficient. Those factors which secure whatever degree of efficiency in the allocation of labor prevails during normal periods cannot be expected to operate as effectively in the short-run when large cyclical changes occur. According to this hypothesis as output falls over the cycle the proportion of output produced by older vintages of equipment, which have higher labor requirements, increases and average labor productivity declines. Without knowledge of both firms' hiring policy and shifts in allocation of labor among vintages of capital over the cycle it is impossible to use data for years of cyclical change to estimate a production function without biasing the result. To use observations over the cycle would have the effect of forcing the parameter estimates to predict short-run changes in apparent productivity that are determined by other than technological factors.

Since the use of data for years of large cyclical change involves the difficulties described above it is necessary to estimate the model with data for stable high employment years only. By this criterion, of the eighteen years of data from 1947 through 1964, only eight are usable: 1947, 1948, 1951, 1952, 1955, 1956, 1963, 1964.⁴ One other period can be added to this list, the second half of 1959 and the first half of 1960, which was the short peak following the 1957-58 and preceding 1960-61 recessions.⁵ It is difficult to assess the degree to which inefficiency due to mis-allocation of labor among vintages of equipment might have existed during this period, but there was likely to have been little hoarding of labor given the length of time that elapsed since the 1958 trough.

Altogether this leaves nine annual observations; but it is not clear how many degrees of freedom are contained in this sample. Several considerations suggest that the number of degrees of freedom is considerably less than is implied by the number of observations. The primary concern is that the appropriate length of the accounting period for the problems at hand may be longer than a year. Lags in the adjustment of employment and in the process of converting investment

4. Although 1963 and 1964 both have comparatively high unemployment rates, the assumption of efficient production is not unwarranted since these years follow two years of steady expansion from the early 1961 trough and little hoarded labor or inefficiency in the allocation of output among vintages of equipment can be expected to have remained from the 1960-61 recession.

5. For those industries that produce and use iron and steel products it was necessary to omit that part of this period during which the steel strike took place.

expenditure into capital-in-use are such that for the dependent variables year to year changes in the deviation from trend values will be largely dominated by short-run or random factors. For these reasons data on employment and output for a pair of consecutive years were averaged and considered as one observation. As a result, only five observations were used: 1947-1948, 1951-1952, 1955-1956, 1959-1960,⁶ and 1963-1964.

3. The shortage of usable data made it necessary to impose an external constraint on the model to obtain estimates of the parameters. In order to proceed the purely arbitrary assumption was made that the rate of capital augmenting disembodied technical progress (r) equals zero.⁷ With five observations there are 10 independent relations (one output equation and one employment equation for each observation) and 10 unknowns (5 technology parameters and the age of marginal vintage for each period of observation). The problem, therefore, is reduced to finding a perfect fit to the data. Solving the 10 equations for the 10 unknowns will yield the set of parameter values which, given the history of investment in equipment, explains all of the observed variance in output and employment.

6. Data for fiscal year only.

7. The results indicate that technical change is relatively labor augmenting. Therefore this constraint turns out to be less costly than constraining either h or s equal to zero, since it is likely that r is closer to zero than either h or s . But as between r or g , there seem little basis to choose which one to set equal to zero.

Due to the nonlinear form of equations (5) and (6) -- and particularly because the $m(t)$ enter in the lower limit to the index of a sum -- a direct approach to the solution of this problem is not feasible. This fact made it necessary to approximate a perfect fit through a trial and error procedure. The results reported in this chapter were obtained by using the following procedure.

A range of plausible values for each parameter, except a_0 and b_0 was defined. For technical change parameters (g , h and s) the range was .00 to .05; for the ages of the marginal vintage (the $m(t)$) the range was 5 to 40. The value of each parameter was varied over its range by small increments -- .001 for g , h , and s , and $1/4$ for the $m(t)$. For each possible parameter set $X = \{g, h, s, m(1), \dots, m(5)\}$ the a_0 and b_0 for each observation were calculated from equations (5) and (6). For any X , errors in a_0 , E_a , and errors in b_0 , E_b , were defined as follows:

$$E_a(i,j) = |a_0(i) - a_0(j)|/a_0(i)$$

$$E_b(i,j) = |b_0(i) - b_0(j)|/b_0(i)$$

$$i = 1, \dots, 5; \quad j = i + 1, \dots, 5$$

For the first X , errors were calculated until an error exceeded a small number E^* , chosen to be .01. If this never occurred, all possible errors were calculated. If any error exceeded E^* the X was discarded; if not the X was stored. In either case the next X was considered. This procedure was followed until all X 's had

been considered. From the set of X's for which all $Ea(i,j)$ and $Eb(i,j)$ were less than E^* , that $X = \hat{X}$ for which

$$\sum_i \sum_j (Ea(i,j)^2 + Eb(i,j)^2)$$

was a minimum was chosen as a solution.

In summary, this procedure chooses as most closely approximating a perfect fit, from among those combinations of parameter values whose maximum percentage error is less than E^* , that combination \hat{X} which has a minimum sum of squared errors. Two points concerning the particular criteria employed to determine \hat{X} should be noted. First, the condition that all errors for a possible solution must be less than E^* was imposed to reduce computation time. Use of this condition makes it possible to reject an X as a possible solution without calculating all 20 errors. However, the possibility remains that an X which has one or more errors greater than E^* , but which has a smaller sum of squared errors than \hat{X} , will be rejected as a solution.

Second, the use of minimum sum of squared errors as a criterion for a solution is completely arbitrary. However, an evaluation of those X 's whose sums of squared errors were close to that of \hat{X} showed none to be superior to \hat{X} under a number of alternative criteria.

It is of some significance that all sets of parameter values which come close to providing a perfect fit to the data show a pattern of results that is substantially the same. Results that are qualitatively different from those reported here do not appear to be

consistent with the data. For this reason these results may be interpreted as giving a reasonably accurate estimate of the structure of the model.

4. The results obtained by fitting equations (5) and (6) to the data are summarized in Table 1.

Since these estimates are the result of a "no degrees of freedom" calculation it is, of course, impossible to measure their statistical reliability.

Table 1. Estimated Parameter Values

<u>Technological Parameters</u>		<u>Age of the Marginal Vintage</u>
Rate of equipment augmentation, embodied	$(g) = .006$	$m(1947-48) = 23$
Rate of labor augmentation, embodied	$(h) = .014$	$m(1951-52) = 22$
Rate of labor augmentation, disembodied	$(s) = .017$	$m(1955-56) = 17$
Output/equipment in 1900	$(ao) = 1.088$	$m(1959-60) = 14$
Output/production workers in 1900	$(bo) = 1.780$	$m(1963-64) = 14$

It is possible to use the results of Table 1, together with data for actual output and production worker employment, for periods of large cyclical change to check the assumption that such periods would be characterized by inefficient production. This can be done by comparing the amount of marginal vintage equipment required to produce actual output efficiently with the amount required to put actual employment to work efficiently. When the latter exceeds the former the existence of "disguised unemployment" is implied. The amount of underutilization of employed production workers can be obtained by multiplying the difference in required amounts of marginal vintage equipment by the per unit production worker requirement. These calculations are shown in Table 2.

Table 2. Efficiency loss during period of large cyclical change.

	equation (5)		equation (6)		production worker
	<u>m</u>	<u>Z</u>	<u>m</u>	<u>Z</u>	<u>disguised unemployment</u>
1947-48	23	1.00	23	1.00	0.0%
1949-50	21	1.00	21	.60	- 1.5
1951-52	22	1.00	22	1.00	0.0
1953-54	19	.20	20	.10	1.5
1955-56	17	1.00	17	1.00	0.0
1957-58	13	.50	13	.90	1.6
1959-60*	13	.70	13	.90	1.2
1961-62	13	.80	13	.90	0.2
1963-64	14	.25	14	.25	0.0

m = age of marginal vintage

Z = proportion of marginal vintage in use

rate of disguised unemployment - production worker requirement
per unit of marginal equipment times the amount of marginal
equipment manned (from equation (6)) less amount of marginal
equipment required (from equation (5)).

*calculation for 1959-60 based on data for the entire two year
period, as opposed to data for fiscal year used in calculations
that underlie the results of Table 1.

The presumption that recession periods would have positive
rates of disguised unemployment is largely borne out by the results
of Table 2. Two exceptions are apparent, however. For 1949-50
instead of actual employment exceeding required employment as
predicted by the model, the calculated rate of disguised unemployment
is negative. One possible explanation is that the unpredicted in-
crease in efficiency from 1947-48 to 1949-50 was due to the
continuation of post-war readjustment and associated productivity
gains into the latter period. An alternative explanation is that the
build-up related to the outbreak of the Korean War brought forth
short lived productivity gains. A second exception is 1961-62 when
disguised unemployment was very small, despite a high rate of un-
employment. Since, for this recession period, most of the decline
occurred in 1960 this suggests that the length of time required to
eliminate disguised unemployment for production workers may be
fairly short.

5. The parameter estimates of equations (5) and (6) shown in Table 1 have three characteristics which are of particular interest. These relate to the bias of technical change, the embodiment of technical change and the interaction of investment and technical change as reflected in the obsolescence of existing capital.

Technical change is incorporated in this model in such a way that it is equivalent to an increase in the quantity of productive factors. The bias of technical change has to do with the relative rates of capital and labor augmentation.⁸ Following Harrod's classification scheme, technical change may be said to be neutral if the rate of capital augmentation is zero and relatively capital augmenting if the rate of capital augmentation is positive. While these definitions are entirely arbitrary, there are a number of reasons why it is useful to consider purely labor augmenting technical change as a condition of neutrality. When technical change is Harrod-neutral balanced growth paths exist for which the rate of growth of output and both augmented inputs is exponential and equal to the natural rate,⁹ and for which both the saving rate and the age of the oldest vintage of capital in use are constant. Furthermore, along these paths both the marginal efficiency of investment and the competitive distribution of output are constant. And, finally, given arbitrary initial conditions and a fixed saving rate, the economy

8. For an analysis of the conditions under which technical change may be described as factor augmenting and a discussion of alternative classification schemes see E. S. Phelps, "Axioms for Factor Augmenting Technical Progress," Cowles Foundation Discussion Paper No. 196.

9. The natural rate of growth is the rate of growth of augmented labor.

will asymptotically converge to a balanced path. And, finally, there is a "golden rule" path, along which consumption is higher than on any other balanced path and the marginal product of investment is equal to the rate of growth.¹⁰

When the rate of capital augmentation is positive none of these results hold. If the rate of growth of output is held constant, then the age of the marginal vintage will be constant, but the saving rate must decline and the marginal product of investment must rise. If the saving rate is fixed, then the age of the marginal vintage and the marginal product of capital will decline. The rate of growth of output will exceed the natural rate of growth of (augmented) labor input and will decline. Eventually the age of the marginal vintage becomes zero and the neoclassical properties of the model no longer hold since no opportunities for shifting labor from older to newer vintages remain.¹⁰

The analytical importance of the bias of technical change is clear, but empirical evidence on the rate of capital augmentation is slight. Since a great deal of production function estimation has made use of the Cobb-Douglas model, for which capital and labor augmentation are equivalent, little effort has been made to identify the bias of technical change. Furthermore, what results do exist have been obtained for models which assume disembodied technical

10. For proofs of these results see Solow, et al, op. cit.

progress and a positive elasticity of substitution.¹¹ There is no reason to expect that these estimates of rates of capital and labor augmentation should be directly comparable to the estimates obtained for a fixed coefficients vintage model.

Unfortunately, the results of Table 1 do not provide sufficient information to determine the extent of factor bias of technical change since they relate only to the equipment and production worker coefficients. The estimate for g indicates only that the rate of equipment augmentation appears to have been 0.6 per cent per year. Similarly, the sum of the estimates for h and s indicate only that the rate of production worker augmentation appears to have been 3.1 per cent per year. In order to determine the overall rate of capital augmentation it is necessary to know both the rate at which the plant requirements per unit of equipment are changing and the rate of plant augmenting technical progress. And, to determine the overall rate of labor augmentation it is necessary to know the rate at which the nonproduction worker requirement changes over time. Estimates of these

11. For example, the CES model with disembodied technical change, assuming perfect competition, implies that the log of the capital-labor ratio is a linear function of the log of the ratio of capital's share to labor's share and time in which the slopes identify the elasticity of substitution and the difference between the rates of factor augmentation. This relationship is estimated in P. A. David and Th. van de Klundert, "Biased Efficiency Growth in the U. S." American Economic Review, 1965, pp. 357-94. The annual rates of augmentation are estimated to be .022 for labor and .015 for capital and the elasticity of substitution is estimated to be .31. These results suggest a considerably higher rate of capital augmentation than a casual look at the data would suggest. See for example the short-hand estimates of Phelps, op. cit., p. 13 which give capital and labor rates of augmentation of .025 and .005 per year.

parameters will be introduced in Chapter III.

6. A second characteristic of technical change that is of special interest is the degree to which it must be embodied in new capital to be effective. The idea that increases in efficiency apply only to new capital was given systematic formulation first by Johansen¹² and later by Solow.¹³ Because of the simplicity of the model in which it was embedded, Solow's version has received extensive analysis. Since Solow's original formulation assumes the production function for each vintage to be Cobb-Douglas with the same exponents for all vintages, technical change can be expressed as purely capital augmenting progress. As a result, if labor is allocated efficiently among vintages of capital, it is possible to construct an aggregate stock of homogeneous capital and retain the Cobb-Douglas form, with the same exponents as in the vintage production function, for the aggregate production function.

In the context of the Solow model the significance of the embodiment hypothesis can easily be overstated. For equivalent rates

12. L. Johansen, op. cit.

13. R. M. Solow, "Investment and Technical Progress," in Mathematical Methods in the Social Sciences, 1959, K. J. Arrow, S. Karlin and P. Suppes, eds. (Stanford, Stanford University Press) 1960.

of technical change,¹⁴ Cobb-Douglas models with and without embodiment have the same asymptotic rate of growth for a constant saving rate. Furthermore, the elasticity of the height of the golden age growth path with respect to the saving rate is identical for the two models.¹⁵ Thus, from a long-run point of view, in the context of a Cobb-Douglas production model, the embodiment hypothesis does not appear to be of great importance.

In the short-run, however, the rate of growth of output is more sensitive to changes in the saving rate for the embodiment model. The higher the rate of investment, the higher will be both the current rate of growth of the capital stock (capital deepening) and the current rate at which the capital stock is modernized (capital quickening). For the embodiment model newer vintages of capital are more productive and the rate of growth of output will depend on both the deepening and quickening effects of current investment.

Since estimation of the production function must be made largely from short-run data,¹⁶ the difference in the short run behavior of the

-
14. The disembodied model takes the form: $Q_t = A_e^{\nu t} K_t^{\alpha} L_t^{1-\alpha}$.
 The embodied model takes the form: $Q = (B \int_{-\infty}^t e^{\lambda v} I_v dv)^{\alpha} L_t^{1-\alpha}$.
 For 'equivalent rates of technical change," $\nu = \alpha\lambda$
15. See E. S. Phelps, "The New View of Investment. A Neoclassical Analysis," Quarterly Journal of Economics, 1962, pp. 548-67.
16. Observations of the economy when it is not on golden age paths.

two models is of considerable importance. This is true even if one is ultimately concerned solely with the long-run implication of the models. For example, if the period of observation is one in which the capital stock is being modernized in excess of the steady state rate, then the estimated disembodied model will indicate more rapid technical change in the steady state than will the embodied model. The embodied model will attribute part of the observed increase in efficiency to the high, but unsustainable, rate of capital quickening. In practice, statistical estimates of the Cobb-Douglas model indicate that the embodied and disembodied versions explain the variation in average labor productivity equally well.¹⁷

In the context of the fixed coefficients vintage model under study here, the embodiment hypothesis assumes crucial importance. If all technical change applies to old and new capital alike then the economy will be able to accommodate only one saving rate and competitive factor pricing will lead to distributive shares of zero and one. Unless there is some embodied technical change, this model reduces to the simple fixed coefficients aggregate capital model described in Chapter I and is incapable of producing the standard neoclassical results.

The estimates reported in Table 1 support the embodiment hypothesis. Since capital augmenting technical change was assumed

17. See R. M. Solow, "Technical Progress, Capital Formation and Economic Growth," American Economic Review, May 1962, pp. 76 - and E. Berglas, "Investment and Technological Change," Journal of Political Economy, 1965, pp. 173-180.

to be purely embodied, only the results for labor augmenting technical change are relevant. These show the embodied and disembodied rates to be 1.4 and 1.7 per cent per year, respectively.¹⁸ The sum of these rates, 3.1, is the rate of growth in production worker efficiency which would occur on a "golden age path," i.e., with a constant age distribution of equipment. It should be noted that if the embodied rate of technical change is constrained to zero, the estimated disembodied rate equals 3.4 per cent per year. The 0.3 difference between this rate and the sum of the embodied and disembodied rates shown in Table 1 reflects, if the embodied model is right, the effect of capital quickening (i.e., reductions in m) over the period of observation.

7 The estimates of the technology parameters shown in Table 1 were obtained simultaneously with, and, therefore, are not independent of, the estimates of the age of the marginal vintage of equipment. The importance of this dependency is made obvious by the fact that for a given sample the age parameters determine the quantity and age composition of equipment in use. A very different set of age parameters would imply a very different set of technology parameters. Clearly, the plausibility of the technology parameters depends crucially on the plausibility of the age parameters.

With respect to both level and trend, the estimated series of the age of the marginal vintage appears to be consistent with what

18. These rates, of course, relate only to production worker augmentation.

little external evidence is available. The only piece of data that can be directly compared to these estimates is the result of a Treasury survey¹⁹ conducted during the late 1950's which indicated the average service life of manufacturing equipment to be 15 years. This is almost identical to the average of the 1955-56 and 1959-60 estimates from Table 1 which cover the same period. The lower figure for the 1959-60 and 1963-64 periods is consistent with the deficient aggregate demand and "excess capacity" that characterized those years. The very high estimates for the early postwar period can be explained by the combination of a very high rate of economic activity relative to "capacity" and the very low rate of investment that prevailed for most of the preceding 15 years.

This apparent decline in the age of the marginal vintage over the postwar period explains, in part, why the estimates of this model support the embodiment hypothesis whereas estimates of the Cobb-Douglas model do not. In order for any model to distinguish between embodied and disembodied technical change, it is necessary that the age structure of the capital stock vary over the period of observation. For the estimation procedure used here, because the age of the marginal vintage is free to be determined by the data, the age structure can vary over a wider range than would be the case if the service life of capital were assumed either to be constant over time or infinite with a constant rate of exponential depreciation. However, the standard measures of capital in use, which underlie the Cobb-Douglas

19. Treasury Depreciation Survey, 1959.

estimates referred to above, do not allow for variation in the rate of obsolescence. For the postwar period this difference in assumption about the useful life of capital results in widely diverging series for the quantity and average age of capital in use. The series implied by the estimates shown in Table 1 and by both a constant service life of 15 years and a constant rate of exponential depreciation are shown in Table 3. Under the assumption of variable service lives, the stock of equipment in use increased over the period as a whole by 50 per cent and its average age declined by three years. By contrast, under the assumption of a constant service life, the stock of equipment in use increased by 130 per cent and its average age increased by 1.7 years, and under the assumption of a constant rate of depreciation of $6 \frac{2}{3}$ per cent the net stock of equipment increased by 95 per cent and the average age increased by 0.3 years. It is clear, for whatever model is being estimated, why neither of the two standard capital series yield results that support the embodiment hypothesis.

On the basis of the results of a simulation study²⁰ it has been suggested that for an economy in which production takes place according to a fixed coefficient vintage model it may be possible to obtain accurate information on such variables as the rate of return on new investment by estimating neoclassical type production functions. What the results of Table 3 indicate, however, is that if the fixed coefficient vintage model gives an accurate representation of the real world it is unlikely that the essential characteristics of production can be obtained by estimating neoclassical models with the kinds of capital stock series that have been used in the past.

20. R. M. Solow, "Heterogeneous Capital and Smooth Production Functions: An Experimental Study," Econometrica, 1963, pp. 623-645.

Table 3. Quantity and Average Age of Equipment in Use.

<u>Period</u>	Variable Service Life (From Table 1)		Constant Service Life (15 years)		Constant Rate of Depreciation (6 2/3% per year)	
	<u>Quantity (bil. 54\$)</u>	<u>Average Age (Years)</u>	<u>Quantity (bil. 54\$)</u>	<u>Average Age (Years)</u>	<u>Quantity (bil. 54\$)</u>	<u>Average Age (Years)</u>
1947-1948	61.2	9.3	43.6	5.3	40.2	8.5
1951-1952	71.9	7.9	60.8	5.9	50.4	8.1
1955-1956	79.5	6.9	75.6	6.4	59.4	8.3
1959-1960	84.3	6.4	90.9	7.0	68.7	8.6
1963-1964	91.5	6.3	100.9	7.0	78.5	8.8

III - OVERHEAD LABOR AND CAPITAL

1. In the preceding chapter the relationships between output and two factors of production, production workers and equipment, were specified and fit to the data. In this chapter relationships involving the remaining two factors of production, plant and nonproduction workers, will be considered. These two sets of relationships are treated independently of one another to offset the substantial increase in complexity that can result when the number of inputs is expanded from two to four in number. To maintain consistency this requires two assumptions regarding technical change which are suggested by the nature of the productive factors and which retain the essence of the model.

The first assumption is that technical change embodied in new equipment does not affect the nonproduction worker requirement per unit of output. This implies that the nonproduction worker requirement per unit of output is independent of the vintages of equipment used to produce that output. Without this assumption it would be necessary to estimate the relationship between nonproduction workers and investment in equipment jointly with the production worker and output relationships. While this would impose no added conceptual difficulties it would involve a prohibitive increase in computation cost. Furthermore, the functions performed

by nonproduction workers¹ suggest, for the most part, that this assumption is correct.²

The second assumption is that technical change embodied in plant is strictly plant augmenting. This implies that the equipment and production worker requirements per unit of output are independent of the vintages of plant that are in use. Without this assumption it would be necessary either to infer from the data or to determine a priori the allocation of equipment of each vintage among different vintages of plant -- an exceedingly complex task. This treatment of plant follows from the view that the function of plant is simply to provide space for other factors of production. Since such items as the installation of lighting, air conditioning, etc., are counted as part of construction expenditure, it is clear that this view cannot be completely accurate. However, since this kind of quality improvement can apply to old as well as new plant, the effects of these changes on the efficiency of other factors of production are likely to appear as disembodied technical change. Thus, for the reasons that the primary function of plant is to satisfy a space requirement and that quality changes that augment other factors may not be restricted simply to new plant, the assumption that technical change embodied in plant is strictly plant augmenting can be considered both reasonable and plausible.

1. Nonproduction workers are engaged in the following activities. Executive, purchasing, finance, accounting, legal, personnel (including cafeteria, medical, etc.), professional and technical activities, sales, sales-delivery, advertising, credit collection, installation and servicing of own products, routine office functions, factory supervision (above the working-foreman level), and force account construction work.

2. However, see p. 36.

2. The data for nonproduction workers and output indicate that the first of these assumptions is correct. In fact, as shown in Table 3, the data indicate that technical change, both embodied and disembodied, has not been nonproduction worker augmenting. It appears that the nonproduction worker requirement is fixed over time at a level of .0324 million workers per one billion dollars of output.

Table 4. Nonproduction Workers and Output.

	1947-48	1951-52	1955-56	1959-60	1963-64
Nonproduction workers (L) millions	2.727	3.286	3.797	4.196	4.539
Output (Q) billions \$	84.1	101.7	115.0	126.1	140.1
$L \div Q$.0324	.0323	.0330	.0333	.0324

Employment of overhead labor exceeded this requirement in two of the five periods observed: by 2.1 per cent in 1955-56 and 2.7 per cent in 1959-60. An obvious explanation for these two observations is hoarding of nonproduction workers either because firms were unable to adjust nonproduction worker inputs to a short-fall of output from expected levels or because firms were maintaining nonproduction worker employment in anticipation of future above-average increases in output. This explanation is suggested by the fact that

for both 1955-56 and 1959-60 the increase in output from the previous peak was considerably less than was the case for the preceding period. Because of the difference in costs of adjustment for the two types of labor, the existence of hoarding of nonproduction workers is not at all inconsistent with the assumed absence of hoarding of production workers.

The relatively slow speed of adjustment in the employment of nonproduction workers in the downward direction is borne out by the data for periods of recession. As shown in Table 5, the rates of disguised unemployment for these periods is considerably higher for nonproduction workers.

Table 5. Rates of Disguised Unemployment

(per cent)

	<u>production workers*</u>	<u>nonproduction workers**</u>
1947-48	0.0	0.0
1949-50	-1.5	-1.2
1951-52	0.0	0.0
1953-54	1.5	3.7
1955-56	0.0	1.9
1957-58	1.6	8.4
1959-60	1.2	6.3
1961-62	0.2	5.9
1963-64	0.0	0.0

* From Table 2

** Assumes that efficient production requires nonproduction worker employment of .0324 times output.

It should be noted that for 1949-50 disguised unemployment for nonproduction workers was negative as it was for production workers. This supports the view expressed in Chapter II that this period was marked by an exogenous but unsustained increase in productivity related perhaps to the build-up for the Korean War.

3. The relationship between output (Q) and nonproduction workers (L) implied by Table 4 may be written as

$$(7) \quad Q(v) = (1.0/.0324) L(v) .$$

Equation (7) together with the estimated output-production worker relationship, which reduces to

$$(8) \quad Q(v) = 1.505 (1.031)^v N(v)$$

when $t = v$, makes it possible to determine the overall rate of labor augmentation. Total labor efficiency may be defined as

$$(9) \quad A(v) = \frac{Q(v)}{L(v)+N(v)}$$

and the overall rate of labor augmentation as $A(t)/A(t)$. Substitution in (9) for $L(v)$ and $N(v)$ from (7) and (8) leaves

$$A(v) = \frac{1}{.0324 + \frac{1}{1.505(1.031)^v}}$$

The overall rate of labor augmentation is

$$(10) \quad \dot{A}(v)/A(v) \approx \frac{.031}{1 + .0486(1.031)^v}$$

which is a decreasing function of v . Equation (10) implies that the overall rate of labor augmentation decreased from 2.6 to 2.3 per cent per year from 1947-48 to 1963-64.

The result that the overall rate of labor augmentation is decreasing depends crucially on the particular way in which total employment is disaggregated. The fact that the nonproduction worker-output ratio was constant over the period of observation implies zero nonproduction worker augmentation and the result described above. This constancy may in fact reflect a peculiarity of manufacturing technology, i.e., that the requirement per unit of output of nonproduction workers, as defined by the Bureau of Labor Statistics, is constant. It may be, however, that the observed constancy is due to the inclusion in the nonproduction worker category groups which have had a particularly rapid rate of growth during the postwar period and which, because of their "equipment-associated" activities, more properly belong in the production worker category. For example, data on industry employment by major occupational group from the labor force household survey³ show that half of the increase from 1952 to 1964 in the aggregate of professional, managerial, clerical, sales and service workers is made up by professionals. If this increase in professionals is related to the modernization of equipment in use, then to include professionals in the nonproduction worker classification constitutes a specification error that may partly

3. Unpublished data, Bureau of Labor Statistics.

explain the zero asymptotic rate of labor augmentation. Clearly, before this result can be accepted, there must be more careful analysis of the disaggregation of employment for the purpose of production function estimation.

4. As indicated in part 1 of this chapter, the estimation of plant requirements is greatly simplified by the assumption that technical change embodied in new plant is strictly plant augmenting. This assumption was suggested by the view that plant is required solely to house production processes. Since production processes are defined in terms of the vintages of equipment in use, the plant requirement at any point in time depends on the quantity and age distribution of equipment in use. Similarly, plant capacity available at any point in time to meet this requirement depends on the quantity and age distribution of plant in use. On the assumption that plant capacity just satisfied plant requirements in each of the five periods of observation the following relationship was fit to data on expenditure for plant (P) and equipment (E) and the estimates of the oldest vintages of equipment in use (m) from Table 1:⁴

$$(11) \quad \sum_{w=0}^{t-n} (1+d)^{t-w} (1+l)^w P(w) = k \sum_{v=t-m(t)}^t (1-e)^v E(v)$$

4. See Chapter II, p.18.

where:

$d \leq 0$ is the rate of depreciation or retirement of plant.

$l \geq 0$ is the rate of plant augmentation embodied in plant.

$e \begin{matrix} < \\ > \end{matrix} 0$ is the rate of decline (embodied in equipment) in
plant requirements per unit of equipment.

$k > 0$ is the amount of plant of vintage 0 required per
unit of equipment of vintage 0 .

$n \geq 0$ is the average time lag from expenditure for plant
to plant in use.

The fitting procedure was identical to that used to calculate the
production coefficients for production workers and equipment.⁵

The results are shown in Table 6.

Table 6. Estimated Parameter Values for Plant-Equipment Relationship.

$d = .026$	$k = .133$
$l = .000$	$n = 2-1/2$
$e = -.014$	

The technology parameters d , l , and e present a somewhat
surprising, but reasonable, picture of the way plant enters the
production process. The estimate of d accords with what little
outside information is available on useful lives of plant. The

5. See Chapter II, pp. 15-17.

depreciation rate of 2.6 per cent per year implies an average life of 38 years for plant. This compares with the figure of 35 years that was indicated by the depreciation survey in the 1950's⁶ and a figure of 40 years given in the 1942 edition of Bulletin F.⁷

The estimated zero rate of plant augmentation implies that after taking into account depreciation a unit of old plant functions as well as a unit of new plant in fulfilling the requirements of any vintage of equipment.

Finally, the estimated value of e indicates that new vintages of equipment have greater plant requirements than older vintages. This may be explained by the need of modern machinery for more expensive space, e.g., space with air conditioning.

Equation (11), together with the output-equipment relationship

$$(2) \quad Q(v) = a(1+g)^v E(v),$$

makes it possible to calculate the overall rate of capital augmentation. Define total capital efficiency as

$$(12) \quad B(v) = \frac{Q(v)}{P(v)+E(v)}.$$

6. Treasury Depreciation Survey, op. cit.

7. Internal Revenue Service.

Substituting in (12) for $P(v)$ and $E(v)$ from (11) and (2) leaves

$$B(v) = \frac{1}{\frac{1}{a} \left(\frac{1}{1+g}\right)^v + \frac{k}{a} \left(\frac{1-e}{1+g}\right)^v} .$$

The overall rate of capital augmentation is given by

$$(13) \quad \dot{B}(v)/B(v) = - \frac{\left[\frac{1}{a} \left(\frac{1}{1+g}\right)^v\right] \ln \left(\frac{1}{1+g}\right) + \left[\frac{k}{a} \left(\frac{1-e}{1+g}\right)^v\right] \ln \left(\frac{1-e}{1+g}\right)}{\frac{1}{a} \left(\frac{1}{1+g}\right)^v + \frac{k}{a} \left(\frac{1-e}{1+g}\right)^v} .$$

Substituting estimated parameter values from Tables 1 and 4 leaves

$$\dot{B}(v)/B(v) \approx \frac{[.92(.994)^v] (.006) + [.12(1.008)^v] (-.008)}{.92(.994)^v + .12(1.008)^v} .$$

That is, the overall rate of labor augmentation is a weighted average of the rates of decline in equipment and plant requirements (.006 and -.008), with the current requirements as the weights. As v increases, since plant requirements rise and equipment requirements fall, the rate of capital augmentation decreases. For 1947-48, the rate of capital augmentation was .0022 per cent per year; for 1963-64, the rate of augmentation was .0016 per cent per year. In sum, the rate of capital augmentation is barely positive and declining slightly. Taking plant and equipment together, technical change appears to be approximately Harrod-neutral.

IV. FACTOR PRICE IMPLICATIONS OF THE MODEL

1. The primary purpose in formulating and estimating an aggregate production function is to identify the terms on which inputs can be converted into output. To this end, as described in Chapters II and III, the model under study here was fit to data on quantities of inputs and output. For a market economy characterized by profit-seeking producers, however, data on factor prices provide an additional source of information on the economy's technological characteristics. This follows, of course, from the fact that technological relationships partly determine the demand relationships for factors of production. Ignorance of the precise way in which nontechnological phenomena, e.g., the degrees of monopoly and monopsony power, influence factor prices makes it impossible to make unambiguous inferences from these data about technology. Nevertheless, it is of considerable importance in evaluating the usefulness of any particular model to know the extent to which it is consistent with factor price data.

Any calculation of market prices implied by a purely technological model must be made conditional on some assumption about the functioning of markets. The simplest assumption to make is that markets are perfectly competitive. If these predicted prices differ radically from market prices, i.e., if the difference between predicted and actual factor prices cannot be explained by an apparent market imperfection, then this may be interpreted to mean that the

model does not accurately capture the economy's technological characteristics.

The production model estimated in this study applies to only one sector of the economy -- manufacturing. Since factors used in manufacturing may, in general, be used in other industries it is not possible to calculate all efficiency prices without knowledge of the technologies of other industries.¹ It is possible, however, given the assumption of perfect competition, to deduce from the estimated technological structure of the manufacturing industry two prices: the rental of the marginal vintage of equipment and the prices of the "dose" of other factors required per unit of the marginal vintage.

If producers of manufactured output are competitors and behave so as to maximize profit, they will operate all vintages of equipment which return a positive rental. It follows from this that a unit of the oldest, least profitable vintage in use will yield just enough revenue to cover the costs of the complementary factors it requires. For the marginal vintage, then, the following relationship will hold.

$$\begin{aligned}
 (14) \quad q(t-m) &= c(t-m) \\
 &= W_N(t) \cdot n(t-m) + W_L(t) \cdot l(t-m) \\
 &\quad + r_p(t) \cdot p(t-m) + IBT(t-m)
 \end{aligned}$$

1. See pages 49-52.

where $q(t-m)$ = output per unit = $1.088 (1.006)^{t-m}$

$n(t-m)$ = production worker requirement per unit

$$= 1.636 \left(\frac{1.006}{1.014} \right)^{t-m} \left(\frac{1}{1.017} \right)^t$$

$\ell(t-m)$ = nonproduction worker requirement per unit

$$= .0324 q(t-m)$$

$p(t-m)$ = plant requirement per unit

$$= .196 (1.014)^{t-m}$$

$IBT(t-m)$ = indirect business tax per unit = $(\text{Tax}/Q(t)) q(t-m)$

$W_N(t)$ = average compensation of production workers

$W_L(t)$ = average compensation of nonproduction workers

$r_p(t)$ = required rental per unit of plant

$c(t-m)$ = total cost per unit

If both the model and the competitive assumption are correct, then (14) says that the rental per unit of marginal equipment, $q(t-m) - c(t-m)$, should be zero and the cost per dose of complementary factors, $c(t-m)$, should equal $q(t-m)$. That is, if the production model and the estimates of its parameters accurately reproduce the technology of the manufacturing sector and the assumption of perfect competition is correct, then equation (14) should hold when market prices are substituted for W_N , W_L and r_p , and estimated production coefficients are substituted for output per unit and requirements of complementary factors per unit of the marginal vintage of equipment. The results of these substitutions are shown in Table 1.

Table 7. Actual and Predicted Factor Cost*

	<u>1947-48</u>	<u>1951-52</u>	<u>1955-56</u>	<u>1959-60</u>	<u>1963-64</u>
1. $t-m$	24	29	38	45	49
2. W_N	3.433	3.800	4.211	4.593	5.115
3. n229	.206	.179	.158	.144
4. W_L	4 609	4 934	5.695	6.426	6.759
5. l041	.042	.044	.046	.047
6. r_p057	.060	.064	.082	.085
7. p274	.294	.333	.367	.388
8. $13T$113	.110	.116	.125	.139
9. Actual cost -- c	1.104	1.120	1.141	1.176	1.266
10. Predicted cost -- q	1.256	1.294	1.366	1.424	1.459
11. $c \div q$88	.86	.84	.83	.84

Lines 9 and 10 in Table 7 indicate that factor costs do not exhaust the product of the marginal vintage of equipment. As shown in line 11, actual factor costs range from 83 to 88 per cent of their predicted value. This may be interpreted to be an indication that product and factor markets do not function as though they were perfectly competitive. If it is assumed that imperfections exist only in the product market, then it is possible to explain these results in terms of a "degree of monopoly" concept. Predicted cost per unit,

* For sources of market data see Appendix.

$q(t-m)$, can be interpreted as the value marginal product of a dose of complementary factors, and actual cost per unit, $c(t-m)$, can be interpreted as the price per dose. If firms act to maximize profit then $c(t-m)$ must also be equal to the marginal revenue product per dose, and line 11 in Table 7 may be interpreted to be the ratio of marginal revenue to price. Defining the degree of monopoly to be

$$1 - \frac{\text{marginal revenue}}{\text{price}}$$

the results of line 11 imply a degree of monopoly on the order of 15 per cent. Since external evidence, e.g., high concentration ratios in manufacturing industries, accords with the finding of a positive degree of monopoly, the model, as specified and estimated in Chapters II and III, appears to be consistent with the evidence from factor price data.

2. From the estimated production coefficients and factor price data, along with an assumption about future factor prices, it is possible to calculate the expected internal rate of return on new equipment. This calculation provides a further check on the plausibility of the model. Unfortunately, the calculated rates of return cannot be compared to a direct observation of actual rates of return. However, since they are calculated ex post, they may be interpreted as required rates of return and may be compared to the cost of capital implied by data from the securities market.

Table 8 shows in line 1 the gross profit, $q(t) - c(t)$, earned per dollar of new equipment in its first year. If it can be assumed that labor and plant earn the value of their marginal product in other industries, then the gross profit figures can be interpreted as the social value marginal product of new equipment and imply a very high social rate of return on investment in equipment. It is interesting to note that these estimates imply a four to five year "pay-back" criterion for new investment which the corporate finance literature indicates is widely used.²

These gross profit figures do not, however, represent the rental earned by investors because they fail to take into account the opportunity cost of the output foregone by not using the complementary factors with additional units of marginal, instead of new, vintage equipment. The first year rental is shown in line 2.

The implied before and after tax rates of return are shown in lines 3 and 4. They depend not just on the first year rental but on succeeding year rentals as well. Since output per unit of equipment is fixed for a given vintage, the rental declines as the cost of the cooperating factors rise. Here it is assumed that the equipment becomes obsolete in 14 years, and that these costs rise exponentially fast enough to reduce the rental (corresponding to line 2) to zero in that time.

2. See, for example, J. Dean, "Measuring the Productivity of Capital," Harvard Business Review, January-February, 1954, pp. 120-130.

Table 8. Return per Dollar of New Equipment

	<u>1947-48</u>	<u>1951-52</u>	<u>1955-56</u>	<u>1959-60</u>	<u>1963-64</u>
1. First year gross profit	.45	.44	.42	.39	.38
2. First year gross rental	.31	.30	.22	.18	.19
3. Rate of return, before taxes	.24	.23	.13	.09	.10
4. Rate of return, after taxes	.12	.12	.07	.04	.05
5. Market cost of capital	.11	.10	.08	.05	.06

Notes to Table 8:

- Line 1. $[q(t) - c(t)]/\text{relative deflator of new equipment.}$
2. $[q(t) - q(t-m)c(t)/c(t-m)]/\text{relative deflator for new equipment.}$
3. Calculated on the assumption that costs per unit rise exponentially at a rate sufficient to reduce the rental to zero in the fourteenth year.
4. Calculated on the assumption of a 50 per cent tax rate and a double declining balance depreciation rate.
5. Earnings-price ratio for industrial common stocks. See Appendix.

The calculated after tax rates of return, line 4, are somewhat lower, particularly for 1959-60 and 1963-64, than might be expected on the basis of either data on the average rate of return on aggregate capital stock or results for the social rate of return reported in other studies. If these results are in fact too low this would suggest that the differences between the production coefficients of new and marginal vintages implied by the results of Table 1 are too low. This could be due to estimated rates of embodied technical change that are too low or estimated ages of marginal vintages that are too low. Alternatively this could indicate that the expected useful life assumed in the calculation is too low. However, when compared with the market cost of capital, line 5, it is not entirely obvious that the calculated required rates of return, line 4, are too low. The cost of capital is measured by the earnings-price ratio for industrial common stock, which is the relevant criterion for a firm that tries to maximize the wealth of its owners, the stockholders.^{3,4} The decline over the postwar period in the required rate of return on equipment suggested by lines 4 and 5 may be explained by the decline

3. See F. Modigliani and M. H. Miller, "The Cost of Capital, Corporate Finance and the Theory of Investment," American Economic Review, June 1958.

4. For the purpose at hand the cost of debt finance can be safely ignored since this cost has already been accounted for in the calculation of the required rate of return on plant, r_p .

in subjective risk as the stability of the economy has become increasingly apparent to investors. Thus, as with prices for other factors, the predicted price for new equipment (line 4) does not appear to be inconsistent with actual market prices (line 5).

3. In parts 1 and 2 of this chapter both types of labor and plant were considered as a group, with a unit defined to be the vector of quantities of these three factors required per unit of marginal vintage equipment. By making assumptions about the market structure it was possible to derive implications from the production model about the price per unit of these factors which was compared to the price per unit obtained from market data. This procedure was followed because, given the impossibility of factor substitution, the market prices of the individual factors may not reflect the marginal products of each factor as implied by the technology of the manufacturing sector considered in isolation. It is possible to identify the output of a unit of marginal vintage equipment as the marginal product in manufacturing of a dose of these factors, yet the separate marginal products for the factors are indeterminant. Consequently it is assumed that market prices reflect the economy-wide marginal productivities, implied by the set of technologies of the several sectors that use these factors.

However, the indeterminacy just described is not a necessary result for this model. If the model were applied to the entire economy rather than just one sector then the marginal products of each of these three factors would be determined. Any factor for which the supply

constraint was not binding would have a zero marginal product; and if there were only one resource constraint that was binding, the entire marginal product of the dose would be attributed to that factor. If factor supplies are not perfectly inelastic, the scarce resources would increase in supply and the others would contract in supply. This would not lead, however, to an equilibrium where all factors earned a positive price unless the technology possessed certain characteristics.

To see why this is so, consider the problem of efficient production in a one sector economy, with a technology as described by this model, in terms of the techniques of linear programming. In this case the objective cost function to be minimized is:

$$C = r_p P + W_L L + W_N N + \sum_{v=t-m}^t i(v)E(v)$$

subject to the constraints:

$$p(t) r_p + \ell(t) W_L + n(t) W_N + i(t) = q(t)$$

.

$$p(t-m) r_p + \ell(t-m) W_L + n(t-m) W_N + i(t-m) = q(t-m)$$

$$r_p > 0; W_L > 0; W_N > 0; i(v) \geq 0, v = t-m, t$$

where $i(v)$ is the rental rate for equipment of vintage v .

In order for all three factors, P , L and N , to have a positive price, all three resource constraints must be binding and there must

be three marginal, or zero profit, vintages. That is $i(t-m-2) = i(t-m-1) = i(t-m) = 0$. This means that each of these vintages is in use but never is used fully. If this is true, then the last three equations involve only the three prices r_p , W_L , and W_N , and determine them. The first $m-3$ equations then determine the rentals on infra-marginal vintages of equipment. The last three equations are:

$$\begin{aligned} p(t-m+2) r_p + (t-m-2) W_L + n(t-m-2) W_N &= q(t-m-2) \\ p(t-m+1) r_p + (t-m-1) W_L + n(t-m-1) W_N &= q(t-m-1) \\ p(t-m) r_p + (t-m) W_L + n(t-m) W_N &= q(t-m) \end{aligned} .$$

In order for the factor prices r_p , W_L and W_N to be strictly positive the production coefficients, p , ℓ , n and q , must satisfy certain conditions. Among these is the condition that the factor requirements per unit of output are neither all increasing nor all decreasing functions of v . That is, for example, if both $\frac{\ell}{q}$ and $\frac{n}{q}$ are lower for newer vintage equipment, then $\frac{p}{q}$ must be higher for newer vintage equipment. Also, no factor equipment may be either constant or proportional to q across vintages.

The estimated production coefficients for manufacturing do not satisfy all of these conditions. The fact that ℓ is proportional to q implies $r_p = W_N = 0$ and $W_L = a/\ell$. This result raises again the question of how the constancy of the nonproduction worker-output ratio is to be interpreted. It suggests that if this model is to be used as a one sector model then the procedure for disaggregating employment

is of crucial importance and that each factor should be included in the model in a more general way than has been allowed for here in the case of nonproduction workers. The condition that embodied rates of factor augmentation do not have the same sign is satisfied by the estimated coefficients. Successive vintages have declining production worker requirements per unit of output but increasing plant requirements per unit of output. This suggests that, if the problem of employment disaggregation can be solved (for example, by not disaggregating at all) the model is capable of predicting positive factor prices even when each process requires more than one complementary factor.

APPENDIX. VARIABLE DEFINITIONS AND DATA SOURCES

The following is a list of the variables used in this study along with definitions and data sources.

Output (Q) - Gross national product originating in manufacturing, 1954 dollars. Data for 1947-1962 from M. L. Marimont, "GNP by Major Industries," Survey of Current Business, October 1962 and September, 1964. The figure for 1964 is constructed from data on national income originating in manufacturing (National Income and Product (NIP) Table I-10), wholesale price indexes for total manufactures and intermediate materials, supplies and components (Bureau of Labor Statistics, Wholesale Prices and Price Indexes) linked to unpublished series for value of shipments and cost of materials, used in double deflation of GNP 1947-1963 (Brookings Econometric Model Project), and linear projections of manufacturing indirect business taxes and capital consumption allowances. Annual observations are adjusted to constant 40 hour work week basis by dividing Q by a work week index -- average weekly hours for production workers (BLS, Employment and Earnings Statistics) divided by 40.

Production workers (N) - Average number of full-time and part-time employees in manufacturing (NIP Table VI -14) multiplied by the ratio of production workers to all employees (BLS, Employment and Earnings Statistics).

Nonproduction workers (L) - Average number of full-time and part-time employees (NIP Tables VI-13 and VI-15) less production workers.

Equipment Investment (E) - Purchases of producers' durable goods, 1954 dollars, by manufacturing establishments (unpublished data, Department of Commerce, 1905-1961, linked to data for 1961-1963 from NIP Table V-12).

Plant Investment (P) - Expenditure for structures, 1954 dollars, by manufacturing establishments (same source as equipment expenditure).

Production worker wage rate (W_N) - Average hourly earnings (BLS, Employment and Earnings Statistics) plus average other compensation per employee (NIP Tables VI-1, VI-2, and VI-14), all divided by work week index and the GNP deflator (same source as output).

Nonproduction worker wage rate (W_L) - Compensation of employees (NIP Table VI-1) divided by nonproduction workers excluding proprietors.

Required rental on plant (r_p) - Estimated depreciation rate (Table 5) plus Moody's Baa corporate bond rate, all times structures deflator (same source as plant investment) divided by GNP deflator.

Market cost of capital - Moody's industrial stocks, earnings per share divided by price per share.